**CONTROL CHARTS FOR VARIABLES:**

Control charts based upon measurable of quality characteristics are called as control charts for variables. Control chart for variables are often found to be more economical means of controlling quality than control charts based on attributes. The variable control charts that are most commonly used are $\bar{X}$, $R-$chart and $\sigma-$charts.

**$\bar{X}-$ CHART:**

When dealing with a quality characteristic that can be expressed as a measurement, it is customary to monitor both the mean value of the quality characteristic and its variability. Control over the average quality is exercised by the control chart for averages, usually called the **$\bar{X}-$ Chart**. Sometimes, it is used alone but very frequently it is paired with a control chart for either range or standard deviation.

$\bar{X}-$ Chart is constructed on the basis of the series of samples drawn frequently during a production process, which are called sub-groups or rational sub-groups of items. Mean value of each sub-group is taken into account under the assumption that the variations within the sub-groups can only be due to chance but if there is significant variation between the various sub-groups, then it indicates the presence of the assignable causes of variations. Then the value of sample mean will differ from sample to another. Since each observation is independent and contributes very little to the total variability of the characteristic (X) under study, so X may be supposed to be normally distributed with mean μ and variance $\sigma^2$, then the distribution of $\bar{X}$ is also normal i.e if $X\sim N(\mu, \sigma^2)$, then $\bar{X} \sim N(\mu, \sigma^2/n)$.

In the choice of control limits , two cases arise:

i) When Specification is given i.e when $\mu$ & $\sigma^2$ are known, then the control limits of $\bar{X}-$ chart are:

$$UCL= E(\bar{X})+ 3S.E(\bar{X}) = \mu + 3\,{}^{\sigma}\!/\!_{\sqrt{n}} = \mu + A\sigma$$

$$CL= \mu \text{ and } LCL= E(\bar{X})-3S.E(\bar{X}) = \mu - 3\,{}^{\sigma}\!/\!_{\sqrt{n}} = \mu - A\sigma, \text{ where } A=3/\sqrt{n}$$

ii) When Specification is not given i.e when $\mu$ & $\sigma^2$ are not known, then few samples are taken and we obtain their means and S.D.

Suppose, we draw k independent samples each of size n from the lot. Let $\bar{X}_1, \bar{X}_2, \ldots\ldots\ldots \bar{X}_k$, be the means of the observations. We define

$$\bar{\bar{X}} = \frac{1}{k}\sum_{i=1}^{k}\bar{X}_i$$

Since E($\bar{\bar{X}}$) $= \mu$, while as estimate of S.D is given by E(s)=$d_2$ σ ,

which gives σ=E (s)/$d_2$= $\bar{s}/d_2$

Therefore, control limits for $\bar{X}$-chart, when specification is not given are:

UCL=$\bar{\bar{X}}$ +3$\bar{s}/d_2\sqrt{n}$ =$\bar{\bar{X}}$+ $A_1\,\bar{s}$, where $A_1 = \frac{3}{d_2\sqrt{n}}$ and $\bar{s} = \frac{1}{k}\sum s_i$

CL= $\bar{\bar{X}}$ and LCL= $\bar{\bar{X}}$- $A_1\,\bar{s}$, The S.D (σ) may be also be estimated from range as:

E(R)= $C_2$ σ, which gives σ=E (R)/$C_2$= $\bar{R}/C_2$, where $\bar{R} = \frac{1}{k}\sum R_i$.

Hence UCL= $\bar{\bar{X}}$+3 $\bar{R}/C_2\sqrt{n}$ = $\bar{\bar{X}}$+ $A_2\,\bar{R}$, where $A_2 = \frac{3}{C_2\sqrt{n}}$

CL=$\bar{\bar{X}}$ and LCL= $\bar{\bar{X}}$-$A_2\,\bar{R}$

The values of A, $A_1$ and $A_2$ are available from tables.

**R-CHART:**

The R-Chart is used as a measure of sub-group dispersion. Process variability can be controlled by either a range chart (*R* chart) or a standard deviation chart (*S* chart), depending on how the population standard deviation is estimated. The importance of R- chart depends on the type of production process. There are many production processes in which it is difficult to maintain uniform process dispersion, in such processes R-chart is extremely useful for process control. It is particularly useful for those processes where the skill of operator is important. The first step in improving such processes should be to try to bring the process dispersion into statistical control.

For drawing R-chart, samples of small size 4 or 5 are drawn randomly from the lot and their ranges are calculated. The sample numbers are taken along x-axis and the value of range along y-axis and points are plotted.

In the choice of control limits, two cases arise:

i)When Specification is given i.e when $\sigma^2$ is known, then the control limits of R − chart are:

UCL= E (R)+ 3 S.E (R) $=$ $d_2\sigma + 3d_3\sigma = D_2\sigma$ , where $D_2=(d_2+3d_3)$

CL= $E(R)$ and LCL= E (R)-3 S.E (R) $=$ $d_2\sigma - 3d_3\sigma = D_1\sigma$ , where $D_1=(d_2-3d_3)$

ii)When Specification is not given i.e when $\sigma^2$ is not known, then its estimate is obtained from k-samples each of size n.

i-e E (R )= $d_2\sigma$, which gives $\sigma$=E (R)/$d_2$= $\bar{R}/_{d_2}$, where $\bar{R}=\frac{1}{k}\sum R_i$

Therefore, control limits for R-chart, when specification is not given are:

UCL= E (R)+ 3 S.E (R) $= \bar{R} + \frac{3\,d_3\bar{R}}{d_2} = \bar{R}\,(1 + \frac{3\,d_3}{d_2}) = D_4\bar{R}$

CL=$\bar{R}$ and LCL= $\bar{R}\,(1 - \frac{3\,d_3}{d_2}) = D_3\,\bar{R}$ The values of $D_1$, $D_2$, $D_3$ and $D_4$ are available from tables. Since range can not be negative, therefore LCL must be greater than zero. If in certain cases it comes out to be negative, then LCL is taken as zero.

When preliminary samples are used to construct limits for control charts, these limits are customarily treated as trial values. Therefore, the *k* sample means and ranges should be plotted on the appropriate charts, and any points that exceed the control limits should be investigated. If assignable causes for these points are discovered, they should be eliminated and new limits for the control charts determined. In this way, the process may be eventually brought into statistical control and its inherent capabilities assessed. Other changes in process centering and dispersion may then be contemplated. Also, we often study the *R* chart first because if the process variability is not constant over time the control limits calculated for the chart can be misleading.

**CONTROL CHART FOR STANDARD DEVIATION (S-CHART):**

When the sample size n is moderately large say n>10, the range method of estimating σ looses statistical efficiency. In these cases, it is best to replace the $\bar{X}$ and R charts by $\bar{X}$ and $S -$ charts, where the process standard is estimated directly instead of indirectly through the use of R. Rather than base control charts on ranges, a more modern approach is to calculate the standard deviation of each subgroup and plot these standard deviations to monitor the process standard deviation _. This is called an S -chart. When an S- chart is used, it is common to use these standard deviations to develop control limits for the chart. Typically, the sample size used for subgroups is small (fewer than 10) and in that case there is usually little difference in the chart generated from ranges or standard deviations. However, because computer software is often used to implement control charts, S -charts are quite common.

For drawing S-chart, sample of small size are drawn randomly from the lot (when the process is in progress) and their sample standard deviations are calculated. Serial No. of samples are taken along X-axis and S.D is taken along Y-axis and points are plotted.

In the choice of control limits, two cases arise:

i)When Specification is given i.e when $\sigma$ is known, then the control limits of $S-\text{chart}$ are:

UCL= E (s)+ 3 S.E (s) $= c_2\sigma + 3c_3\sigma = B_2\sigma$ , where $B_2=(c_2+3c_3)$

CL= $E(s)$ and LCL= E (s)-3 S.E (s) $= c_2\sigma - 3c_3\sigma = B_1\sigma$ , where $B_1=(c_2-3c_3)$

ii)When Specification is not given i.e when $\sigma$ is not known, then its estimate is obtained from k-samples each of size n.

i-e E (s )= $\frac{1}{k}\sum s_i$, which gives $\sigma$=E (s)/$c_2$= $\bar{C}/c_2$, where $\bar{s}=\frac{1}{k}\sum s_i$

Therefore, control limits for S-chart, when specification is not given are:

UCL= E (s)+ 3 S.E (s) $= \bar{s} + \frac{3\,c_3\bar{s}}{c_2} = \bar{s}\,(1 + \frac{3\,c_3}{c_2}) = B_4\bar{s}$

CL=$\bar{S}$ and LCL= $\bar{s}\,(1 - \frac{3\,c_3}{c_2}) = B_3\bar{s}$

The values of $B_1$, $B_2$, $B_3$ and $B_4$ are available from tables.